5.1 Duality

$$\forall x A(x) \leftrightarrow \neg \exists x \neg A(x) \quad \exists x A(x) \leftrightarrow \neg \forall x \neg A(x)$$

5.2 Commutativity and Distributivity

$$\forall x \forall y A(x,y) \leftrightarrow \forall y \forall x A(x,y) \quad \exists x \exists y A(x,y) \leftrightarrow \exists y \exists x A(x,y)$$
$$\exists x \forall y A(x,y) \rightarrow \forall y \exists x A(x,y)$$

$$\models \forall x A(x) \vee \forall x B(x) \rightarrow \forall x\,(A(x) \vee B(x))$$
$$\models \exists x\,(A(x) \wedge B(x)) \rightarrow \exists x A(x) \wedge \exists x B(x)$$

Suppose $D = \{d_1, d_2\}$, and

$$v\,(A(d_1)) = T, \qquad v\,(A(d_2)) = F,$$
$$v\,(B(d_1)) = F, \qquad v\,(B(d_1)) = T.$$

Then, $v\,(\exists x A(x) \wedge \exists x B(x)) = T$, but $v\,(\exists x\,(A(x) \wedge B(x))) = F$.

5.3 Quantification Without the Free Variable in its scope

$$(\exists x A(x) \vee B) \leftrightarrow \exists x (A(x) \vee B) \quad (\forall x A(x) \vee B) \leftrightarrow \forall x (A(x) \vee B)$$

$$(B \vee \exists x A(x)) \leftrightarrow \exists x (B \vee A(x)) \quad (B \vee \forall x A(x)) \leftrightarrow \forall x (B \vee A(x))$$

$$(\exists x A(x) \wedge B) \leftrightarrow \exists x (A(x) \wedge B) \quad (\forall x A(x) \wedge B) \leftrightarrow \forall x (A(x) \wedge B)$$

$$(B \wedge \exists x A(x)) \leftrightarrow \exists x (B \wedge A(x)) \quad (B \wedge \forall x A(x)) \leftrightarrow \forall x (B \wedge A(x))$$

5.4 Quantification over Implication and Equivalence

$$\forall x(A \to B(x)) \leftrightarrow (A \to \forall x B(x))$$
$$\forall x(A(x) \to B) \leftrightarrow (\exists x A(x) \to B)$$

$$(\exists x(A(x) \vee B(x)) \leftrightarrow (\exists x A(x) \vee \exists x B(x))$$
$$\forall x(A(x) \wedge B(x)) \leftrightarrow (\forall x A(x) \wedge \forall x B(x))$$
$$\forall x A(x) \vee \forall x B(x)) \to \forall x(A(x) \vee B(x))$$
$$\exists x(A(x) \wedge B(x)) \to (\exists x A(x) \wedge \exists x B(x))$$

$$\forall x(A(x) \leftrightarrow B(x)) \to (\forall x A(x) \leftrightarrow \forall x B(x))$$
$$\forall x(A(x) \leftrightarrow B(x)) \to (\exists x A(x) \leftrightarrow \exists x B(x))$$

$$\exists x(A(x) \to B(x)) \leftrightarrow (\forall x A(x) \to \exists x B(x))$$
$$(\exists x A(x) \to \forall x B(x)) \to \forall x(A(x) \to B(x))$$

$$\forall x(A(x) \vee B(x)) \to (\forall x A(x) \vee \exists x B(x))$$
$$\forall x(A(x) \to B(x)) \to (\forall x A(x) \to \forall x B(x))$$

$$\forall x(A(x) \to B(x)) \to (\exists x A(x) \to \exists x B(x))$$
$$\forall x(A(x) \to B(x)) \to (\forall x A(x) \to \exists x B(x))$$

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