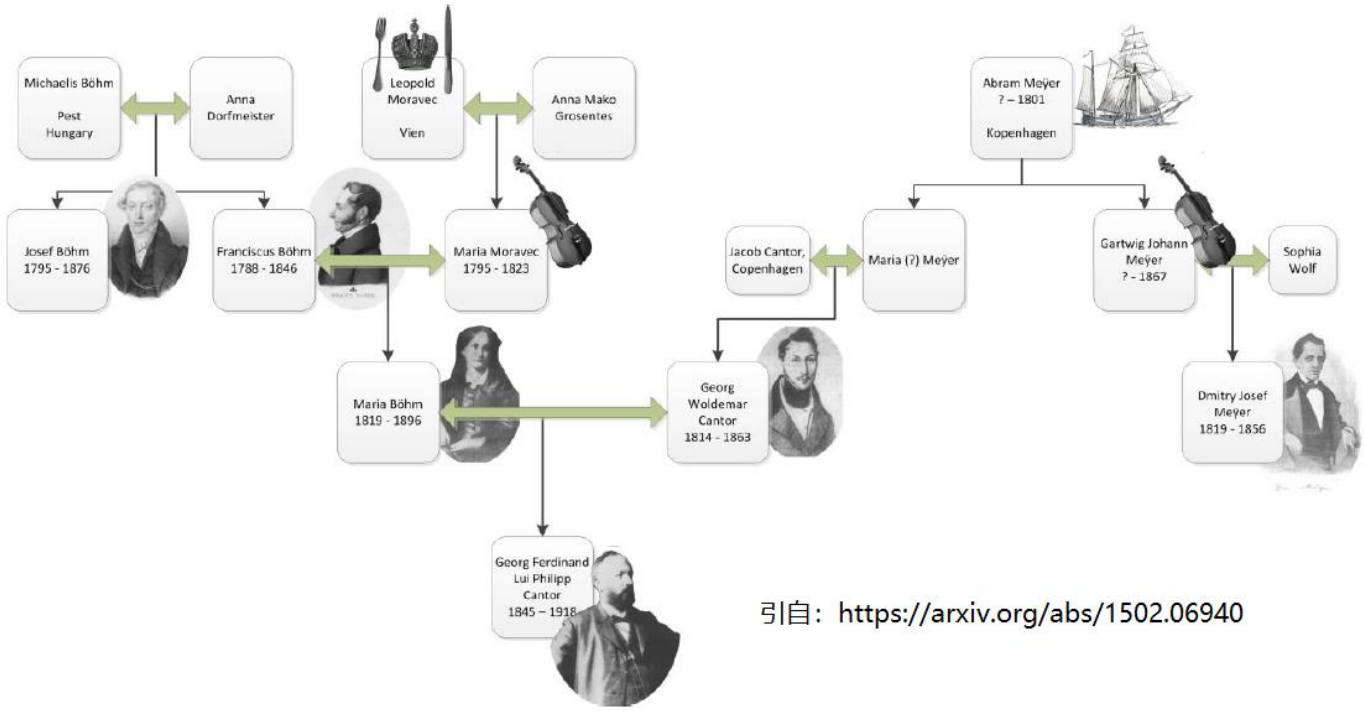
天才数学家-康托

生平

- 康托（Georg Cantor）1845年出生于俄罗斯。父亲是一个很富的商人，在很多国家都有产业；母亲是一个小有成就的音乐家。所以康托很有音乐天赋，后来也是一个优秀的小提琴家。



引自：https://arxiv.org/abs/1502.06940

- 康托在德国柏林大学（University of Berlin）接受教育，上过魏尔斯特拉斯（Weierstrass）、库默尔（Kummer）和克罗内克（Kronecker）的课，在哥廷根大学也上过一个学期的课。之后，康托在哈雷大学（University of Halle）成为教授，那一年他34岁。

Cantor in the 1870s

引自：Dauben J.W. (2005) The Battle for Cantorian Set Theory. In: Van Brummelen G., Kinyon M. (eds) Mathematics and the Historian’s Craft. CMS Books in Mathematics. Springer, New York, NY.

- 在哈雷大学，康托的研究兴趣从数论转到分析，与戴德金（Dedekind）是朋友。

HISTORIA MATHEMATICA **20** (1993), 343–363

On the Relations between Georg Cantor and Richard Dedekind

JOSÉ FERREIRÓS

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Cantoblanco, E-28049 Madrid, Spain

This paper gives a detailed analysis of the scientific interaction between Cantor and Dedekind, which was a very important aspect in the history of set theory during the 19th

- 康托提出可数集合不可数集（Countable and Uncountable Sets）的概念和一一映射（One-to-One Correspondence）的概念，并且得到一个令自己都目瞪口呆的发现: [0,1] 与R^n（对于任意的$n \geq 1$）之间存在一一映射。

THE INVARIANCE OF DIMENSION: PROBLEMS IN THE EARLY DEVELOPMENT OF SET THEORY AND TOPOLOGY [1]

BY JOSEPH W. DAUBEN
HERBERT H. LEHMAN COLLEGE, CITY UNIVERSITY OF NEW YORK

SUMMARIES

In 1878 Georg Cantor proved that unique, one-to-one mappings could be constructed between spaces of arbitrary yet different dimension. This paper is devoted to a detailed analysis of the earliest attempts to deal with the impli-

- 紧接着，康托全面创立集合论，并提出基数（Cardinal Numbers）的概念和对角证明法（Diagonal Method）。可惜的是，当时提出的“连续统”假设连自己也未能证明。

> Just since our recent meetings in Harzburg and Eisenach [Sept. 1882], God Almighty saw to it that I attained the most remarkable and unexpected results in the theory of manifolds and the theory of numbers, or rather that I found what fermented in me for years and what I have long been searching for.

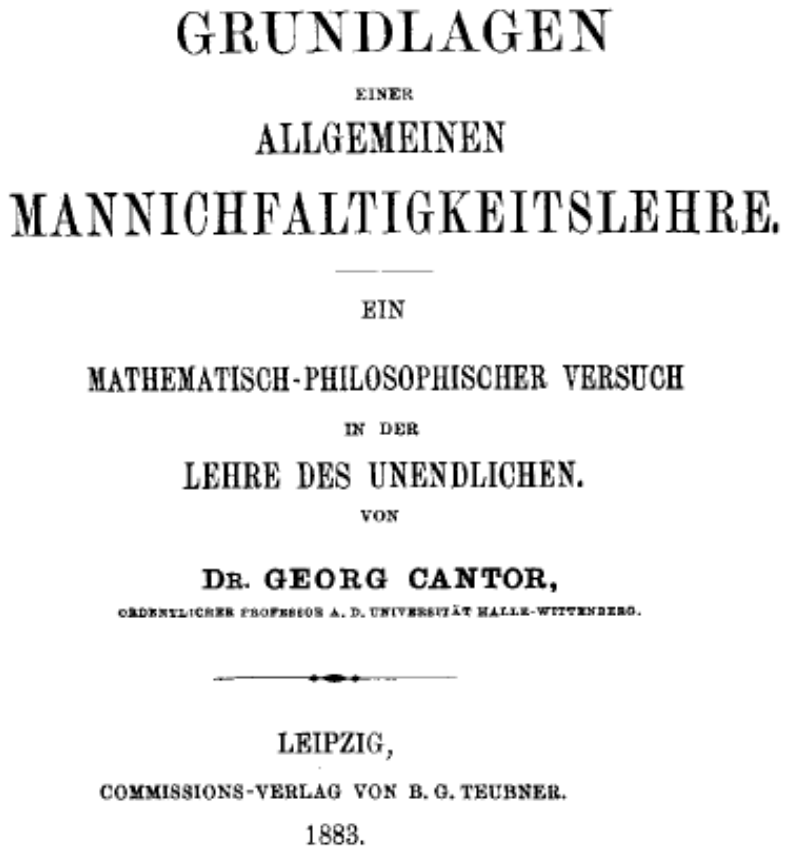
引自：José Ferreirós, “What fermented in me for years”: Cantor's discovery of transfinite numbers, Historia Mathematica, Vol 22, 1, 1995, 33-42.

- 那时候教授收入不高，加上有6个孩子，所以康托晚年贫困，依靠父亲的遗产勉强度日。康托晚年还精神失常，或许有两个诱因: 一个是他未能证明连续统假设，另一个是受到权威数学家的批判。
- 1918年康托在哈雷逝世，享年73岁。

引自:
https://arxiv.org/abs/1502.06940

贡献

- 基数、无限基数的概念



GRUNDLAGEN

EINER

ALLGEMEINEN

MANNICHFALTIGKEITSLEHRE.

EIN

MATHEMATISCH-PHILOSOPHISCHER VERSUCH

IN DER

LEHRE DES UNENDLICHEN.

VON

DR. GEORG CANTOR,

ORDENTLICHER PROFESSOR A. D. UNIVERSITÄT HALLE-WITTENBERG.

LEIPZIG,

COMMISSIONS-VERLAG VON B. G. TEUBNER.

1883.

- 对角证明法

Here is an interesting quote by the logician Wilfrid Hodges:

I dedicate this essay to the two-dozen-odd people whose refutations of Cantor's diagonal argument have come to me either as referee or as editor in the last twenty years or so. Sadly these submissions were all quite unpublishable; I sent them back with what I hope were helpful comments. A few years ago it occurred to me to wonder why so many people devote so much energy to refuting this harmless little argument—what had it done to make them angry with it? So I started to keep notes of these papers, in the hope that some pattern would emerge. These pages report the results.

引自：Lipton R.J., Regan K.W. (2013) Georg Cantor: Diagonal Method. In: People, Problems, and Proofs. Springer, Berlin, Heidelberg.

- 康托分形（Cantor Fractal）

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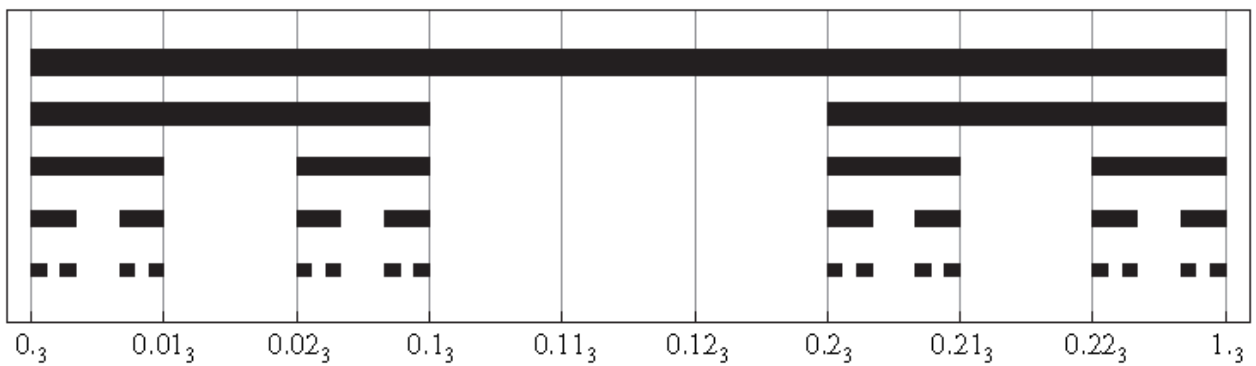

引自：Wagon S. (2010) The Cantor Set, Real and Complex. In: Wagon S. (eds) Mathematica in Action. Springer, New York, NY.

争议

- 庞加莱（Henri Poincare）: “Later generations will regard set theory as a disease from which one has recovered”

One of the most colourful quotes in all the history of mathematics is attributed to Poincaré: "Later generations will regard *Mengenlehre* as a disease from which one has recovered." The quotation is perhaps a little spoiled by the intrusion of the German word for set theory, but doesn't that give it just the aura of verisimilitude required for maximum conviction? How fortunate to have an illustration of those vigorous expressions of opinion we all know mathematicians make in private but seldom confide to the page; how fortunate

引自：Gray, J. Did poincaré say set theory is a disease?. The Mathematical Intelligencer 13, 19-22 (1991).

- 克罗内克（Leopold Kronecker）: “I don’t know what predominates in Cantor’s theory - philosophy or theology, but I am sure that there is no mathematics there”

he was certain mathematics would one day be cured.[1] Leopold Kronecker, one of Cantor’s teachers and among the most prominent members of the German mathematics establishment, even attacked Cantor personally, calling him a “scientific charlatan,” a “renegade,” and a “corrupter of youth.”[2]

引自：Dauben J.W. (2005) The Battle for Cantorian Set Theory. In: Van Brummelen G., Kinyon M. (eds) Mathematics and the Historian’s Craft. CMS Books in Mathematics. Springer, New York, NY.

- 好在有希尔伯特（David Hilbert）的充分肯定: “No one shall expel us from the Paradise that Cantor has created”.

> Indeed, Cantor’s proof that $\mathbf{R}$ is uncountable occupies another spot on the short list of the most significant contributions toward understanding the mathematical infinite. In the words of the mathematician David Hilbert, “No one shall expel us from the paradise that Cantor has created for us.”

引自：Abbott S. (2015) Basic Topology of R. In: Understanding Analysis. Undergraduate Texts in Mathematics. Springer, New York, NY.

名言

- “In mathematics, the art of proposing a question must be held of higher value than solving it.”– Georg Cantor

A straightened proof for the uncountability of $\mathbb{R}$

Andreas M. Hinz

Andreas M. Hinz received his doctoral degree and his habilitation in mathematics from the University of Munich (LMU), where he is now “außerplanmäßiger Professor” in addition to his current professorship for Applied Mathematics at the FernUniversität in Hagen. His main fields of research are real analysis, the history of science, mathematical modelling, and discrete mathematics.

In re mathematica ars proponendi quaestionem pluris facienda est quam solvendi.

G. Cantor, 1867 [8, p. 31]

引自：Hinz Andreas: A straightened proof for the uncountability of $\mathbb{R}$. Elem. Math. 65 (2010), 26–28.