课程简介

课程名称: 高级数理逻辑

课程代码: Comp520

授课教师: 李辉

2021年研究生在线课程建设项目

逻辑（Logic）

- 既是数学的分支，又是哲学的分支。（因为在很多大学两个系都会开逻辑课）

2011-2012 Courses

Harvard

Fall 2011

- Philosophy 143r: Topics in Logic: Proseminar (Koellner)
- Mathematics 141: Introduction to Mathematical Logic (Sacks)
- Computer Science 121. Introduction to Formal Systems and Computation (Lewis)

引自：http://logic.harvard.edu/courses.php （2021）

- 还是计算机分支。（因为推演系统、递归思想等对计算机领域同样非常重要）
- 交集包括但不限于：语言、语义、推演系统等。
- 哲学领域的逻辑会偏向于认识论(Epistemology).
- 数学和计算机领域的逻辑偏向于符号逻辑和形式语言(Formal Language).

逻辑的形式（Logic Form）

哲学领域与数学领域的逻辑的重要差别之一：自然语言(Natural Language)与形式语言。

- 莱布尼茨（Leibniz）认为自然语言表述模糊，应该用形式语言取而代之。

formulated in the object-language.

Before dealing with these principles specifically, I shall consider Leibniz' own formulation of Leibniz' Law. Though he gave various formulations in various places, the following is particularly explicit: "things are the same or coincident which can be mutually substituted, the one for the other, without loss of truth"[3]. Now, as shown by Benson Mates in his article, "Leibniz on Possible Worlds," Leibniz' Law may be formulated either in the metalanguage or in the object-language (or in both). Indeed, Leibniz' formulation seems to involve a use-mention confusion, which the stricter formulations avoid. However, according to Mates, it is not clear whether Leibniz

引自： Friedman, J.I. Plato's Euthyphro and Leibniz' Law. Philosophia 12, 1–20 (1982).

- 有些科学家认为可以考虑把自然语言改造得更严谨一些。
- 还有些科学家认为符号逻辑可以补充自然语言的不足，形成互补。

这种分歧的后果包括：同样一个具体的逻辑，既称谓词逻辑（Predicate Logic），又称一阶逻辑。

数理逻辑（Mathematical Logic）

形式化逻辑系统（Formal Logical Systems， 又称Classical Logical Systems）。

1. 命题逻辑（Propositional Logic）
2. 一阶逻辑（First-Order Logic）
3. 二阶逻辑（Second-Order Logic）
4. 高阶逻辑（Higher-Order Logics）

四个主要子领域:（Jon Barwise的观点）

1. 集合论（Set Theory）
2. 模型论（Model Theory）
3. 递归论（Recursion Theory）
4. 证明论（Proof Theory）与构造性数学（Constructive Mathematics）

各自的关注点相对独立，但相互间以及与其它学科间的界限并不是非常分明。

- 比如，这四个领域的成果往往集中反映在一些经典逻辑主题上，如一阶逻辑。
- 又比如：哥德尔（Gödel）的非完备性定理（Incompleteness Theorem）既是递归论的里程碑，也是证明论的里程碑。

发展大事记

- 古典逻辑在古中国（公孙龙的“白马非马”）、古印度、古希腊和古伊斯兰国家都有记载。

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先秦哲学对属性与本质的思考
——以“白马论”为中心的讨论 *

- 18世纪中期，莱布尼茨率先提出符号逻辑的想法。
- 19世纪中叶，布尔（Boole）和德摩根（De Morgan）进一步系统地用数学方法来研究逻辑。
- 然而从古典逻辑迈向现代逻辑的转折点，被认为是德国哲学家兼数学家弗雷格（Frege）关于形式系统的一本书《Begriffsschrift（概念文字）》。不仅在逻辑中引入数学，弗雷格还提出了用逻辑研究数学基础的思路。在这个方面，罗素（Russell）也做出重要贡献。

- 算术、几何和数学分析的公理化（Axiomatization），主要工作是皮耶罗（Peano）公理，罗巴切夫斯基（Lobachevsky）的平行公设研究，希尔伯特（Hilbert）的几何公理化。公理化的工作一致持续到20世纪的初期。

1. Zero is a number.
2. The successor of any number is another number.

[5] This formulation was given by PEANO in 1898: see abstract in *Opere scelte*, **3**: 215–231, p. 216. Translation from KENNEDY, "The Mathematical Philosophy of Giuseppe Peano," p. 262.

引自: Segre, M. Peano's axioms in their historical context. Arch. Hist. Exact Sci. 48, 201－342 (1994).

- 19世纪末到20世纪初还有一个重要进展是：康托提出来无限集合论，连续统假设，康托定理等。此外，策梅洛（Zermelo，选择公理的提出者）在集合论方面也做出杰出贡献。
- 20世纪初在集合论和公理化领域出现大量悖论和不一致性，如罗素悖论和理查德（Richard）悖论。勒文海姆（Löwenheim）和斯科伦（Skolem）提出勒文海姆–斯科伦定理，这个定理的另一种表现是斯科伦悖论。金琛（Gentzen）在解决悖论和研究一致性方面做了重要贡献，但做出关键贡献的还是哥德尔。有趣的是：斯科伦和哥德尔的工作还形成了模型论的基础，而同时金琛和希尔伯特的工作形成了证明论的基础。

Skolem's paradox was thought by Skolem, and has been thought by his skeptical successors since, to show that the notion of absolute non-denumerability is ineffable. In Skolem's own words (1922):

. . . auf axiomatischer Grundlage sind höhere Unendlichkeiten nur in relativem Sinne vorhanden.

(on axiomatic foundations higher infinities occur only in a relative sense.)

引自： McCarty, C., Tennant, N. Skolem's paradox and constructivism. J Philos Logic 16, 165－202 (1987).

A Second Pick of G

Among von Neumann's audience in 1930 there had very likely been the young student Gerhard Gentzen (1909–1945) who in a letter of 13 December 1932 to his first university professor Hellmuth Kneser wrote:

> I have set as my specific task to find a proof of the consistency of logical deduction in arithmetic… The task becomes a purely mathematical problem through the formalization of logical deduction. The proof of consistency has been so far carried out only for special cases, for example, the arithmetic of the integers without the rule of complete induction. I would like to proceed further at this point and to clear at least arithmetic with complete induction. I am working on this since almost a year and hope to finish soon, and would then present this work as my dissertation (with Prof. Bernays).

This task required as a preliminary a study of logical deduction itself, which Gentzen cleared by his development of the proof systems of natural deduction and sequent calculus by May 1933. It took another 2 years for him to figure out and

引自：von Plato, J. G　del, Gentzen, Goodstein: The Magic Sound of a G-String. Math Intelligencer 36, 22 - 27 (2014)

- 20世纪中科恩（Cohen）提出来Forcing的概念，并证明了连续统假设和选择公理与策梅洛集合论是相互独立的，因此获得菲尔茨奖。
- 图灵和他的图灵机是递归论的核心。哥德尔的证明用到函数的递归定义，便与图灵机密切相关。同时期的邱奇（Church）、克莱尼（Kleene）和波斯特（Post）在递归论领域也做出突出贡献。

Sacks	Didn't you have some feeling of getting the recursion theorem too?
Kleene	I think I got the recursion theorem just a little bit later than that. I got the recursion theorem before I left Princeton in June of 1935, and of course we already had Church's thesis in the late spring of '34 — that is when Church was talking with Gödel about his general recursive functions.

引自：Crossley J.N. (1975) Reminiscences of logicians. In: Crossley J.N. (eds) Algebra and Logic. Lecture Notes in Mathematics, vol 450. Springer, Berlin, Heidelberg.

- 有一些事件表面无关，但实则与数理逻辑密切相关，如：20世纪三十年代法国创办“布尔巴基（Nicolas Bourbaki）合作者协会”，倡导用最严格的风格重写数学书，一度风靡。

contain an amazingly large number of contradictory statements. And even if you pose the question to respected mathematicians, they will reply with such a flurry of amusing anecdotes that, utterly confused, you will be forced to ask yourself in all seriousness: “Does Nicolas Bourbaki really exist? ”

To try to put an end to this confusion, we turn to a man in whom we can have the utmost confidence: Prof. Dr. Gottfried Köthe, former rector of the Gutenberg Universi-

引自：Cartan, H. Nicolas Bourbaki and contemporary mathematics. The Mathematical Intelligencer 2, 175–180 (1980).

考核权重：

- 作业– 20%, 课堂表现 – 10%, 期末考试 – 70%.

教材:

- Michael Huth等著，《面向计算机科学的数理逻辑系统建模与推理》(影印版). 机械工业出版社. 2005. (ISBN 7-111-16053-3).

参 考 书:

- H. D. Ebbinghaus, J. Flum and W. Thomas. Mathematical logic. Springer-Verlag New York Berlin Heidelberg Tokyo. 1985. (ISBN 3-540-90895-1).

以课堂授课内容和讲义为主.